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HYDROGASDYNAMICS IN TECHNOLOGICAL PROCESSES

ON THE INTERACTION OF A VORTEX PAIR AND A VORTEX RING WITH A FLAT SHIELD

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The approximate method of calculation of nonstationary flow in the interaction of a vortex pair and a vortex ring with a parallel and respectively perpendicular flat shield is presented. It is shown that these primary vortices induce transverse wall flow on the shield in the ideal-fluid approximation; in this flow, with allowance for the fluid's viscosity, a boundary layer is generated which represents vortex flow with sign opposite to that of the primary vortices. Boundary-layer separations occur on the portion of the shield with a positive longitudinal pressure gradient. Secondary flows interact with the primary ones due to which the flow is rearranged; the transverse displacement of the initial vortex pair with a loop-shaped trajectory of its motion is observed in the plane problem, whereas the formation of ascending flow along the axis of the vortex ring is observed in the axisymmetric problem. The effect found in the latter case in laminar and turbulent regimes of flow is confirmed for the laminar regime by experiment and by the data of numerical simulation of Navier–Stokes and Reynolds equations.

Keywords: *vortex pair, vortex ring, flat shield, discrete-vortex method, boundary layer, flow separation, experiment.*

Introduction. The discrete-vortex method provides great scope for solution of various problems in the theory of motion of an ideal fluid. However, when separated flows are investigated it is required that viscous effects be taken into account in a number of cases; this brings about the necessity of synthesizing nonviscous potential flow and allowing for the influence of the medium's viscosity within the framework of the nonstationary boundary layer [1]. This is also true of the class of problems in which vortex flow of an ideal fluid induces secondary wall flow accompanied by the formation of a nonstationary boundary layer. On separation of the latter, secondary vortices form which interact with the primary ones calculated within the framework of the ideal fluid.

Below, we consider a plane problem on interaction of a vortex pair with a parallel flat shield. Two parallel vortex bundles located at a fixed distance from the shield at the initial instant of time are investigated and secondary flows due to the medium's viscosity are studied. This problem is of fundamental and practical interest. In particular, it can be used in developing the method of calculation of the vortex wake of an airplane under takeoff–landing conditions [2].

Formulation of the Problem. Two parallel vortex bundles with circulations Γ_0 and $-\Gamma_0$ are located in parallel to a flat shield at distance H_0 from it; the distance between them is $2z_0/H_0 = 1$ (Fig. 1a). For the nonflow condition on the shield to be fulfilled we place two mirror-image vortex structures with circulations whose sign is opposite to the sign of the initial vortex bundles for $y > 0$ at a distance $y = -H_0$ from the shield. The initial vortex bundles and their mirror companions are replaced by systems of 19 rectilinear vortex lines with identical circulations; the circular core is replaced by the central linear vortex and two concentric layers consisting of 6 and 12 identical vortices respectively (Fig. 1c). These vortex bundles induce transverse wall flow at the shield, which is accompanied by the formation of a turbulent boundary layer at large Reynolds numbers. On separation of this layer, secondary vortices having circulation of opposite sign are formed on a portion of the positive pressure gradient in the transverse direction. The secondary vortices induce the transverse displacement of the primary vortices, which follows a loop-shaped trajectory.

The problem is solved in the quasistationary approximation. We use the integral relation of momentum

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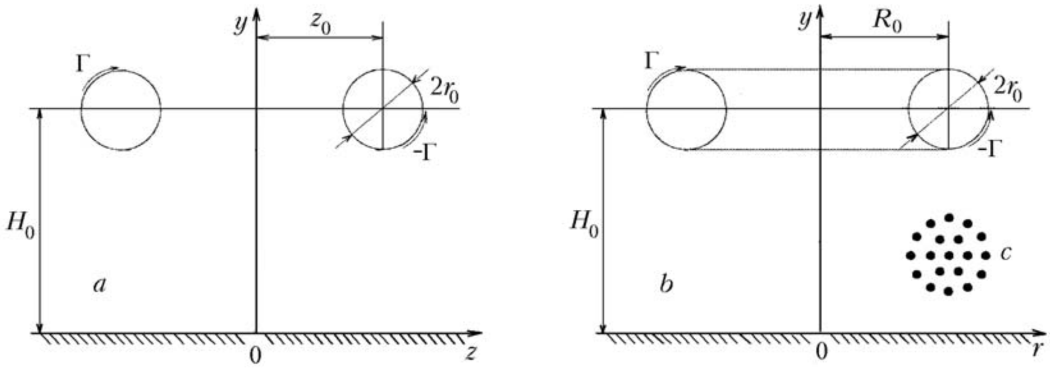

Fig. 1. Diagram of a vortex pair (a) and a vortex ring bundle (b) near the shield; c) representation of the vortex bundle by 19 vortex filaments.

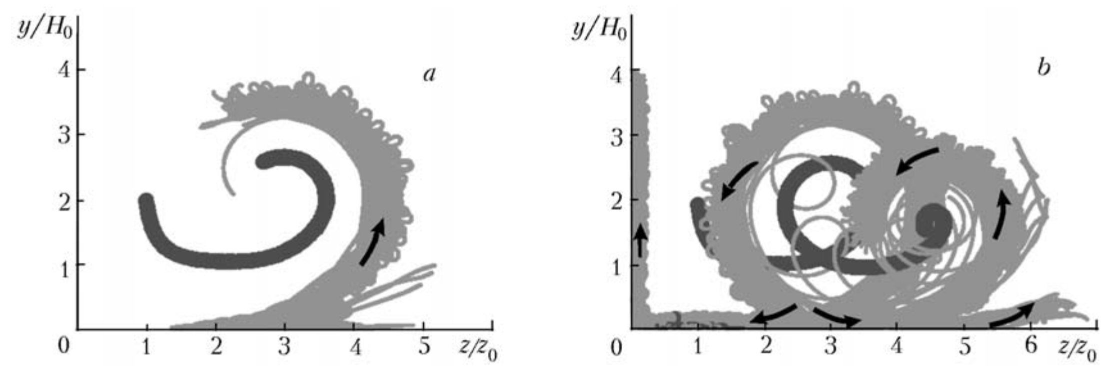

Fig. 2. Action of secondary vortices on the transverse displacement of primary vortices near a flat shield; the trajectories of motion of the primary and secondary vortices at different instants of time (a) $\overline{t}$ = 75 and b) 50). Dark portions correspond to the position of the primary vortices; light portions correspond to that of the secondary vortices.

$$\frac{dR_2}{d\overline{z}} + \frac{1}{\overline{w}_{z\delta}} \frac{d\overline{w}_{z\delta}}{d\overline{z}} (H+1) R_2 = \frac{1}{2} \mathrm{Re}\, \overline{w}_{z\delta} c_{\mathrm{f}} , \tag{1}$$

where $\overline{z} = z/H_0$, $\overline{w} = w_{z\delta}/V_0$, $\mathrm{Re} = V_0 H_0/\nu$, $R_2 = w_{z\delta}\delta_2/\nu$, $H = \delta_1/\delta_2$, and $c_{\mathrm{f}} = \tau_{\mathrm{w}}/\frac{1}{2}\rho w_{z\delta}^2$.

The integral relation (1) contains three unknown parameters R_2, H, and c_{f}. The existing relations for the velocity profile in the boundary layer and the resistance law, i.e., for H and c_{f} [3], should be used to close the problem. The position of the separation cross section is determined from the condition $c_{\mathrm{f}} = 0$. The vorticity of the separated boundary layer in the separation cross section and accordingly the circulation of the separated vortex are determined from the existing expressions

$$\frac{d\Gamma}{dt} = \int_0^{\delta} \frac{\partial w_z}{\partial y} w_z dy = \frac{1}{2} w_{z\delta}^2 , \quad \Gamma_{\mathrm{s}} = \frac{1}{2} w_{z\delta}^2 \Delta t . \tag{2}$$

The last relation is used in calculating the circulations of the secondary vortices at different instants of time.

The algorithm of numerical solution of the primary vortices is constructed as follows. First we calculate, on the time interval [0, Δt], the dynamics of the primary vortices within the framework of the ideal fluid and determine the velocity $w_{z\delta}$ at the boundary of the boundary layer (more precisely, on the shield) at the end of the interval. Next we solve the integral momentum relation for the boundary layer up to the coordinate of the point of separation

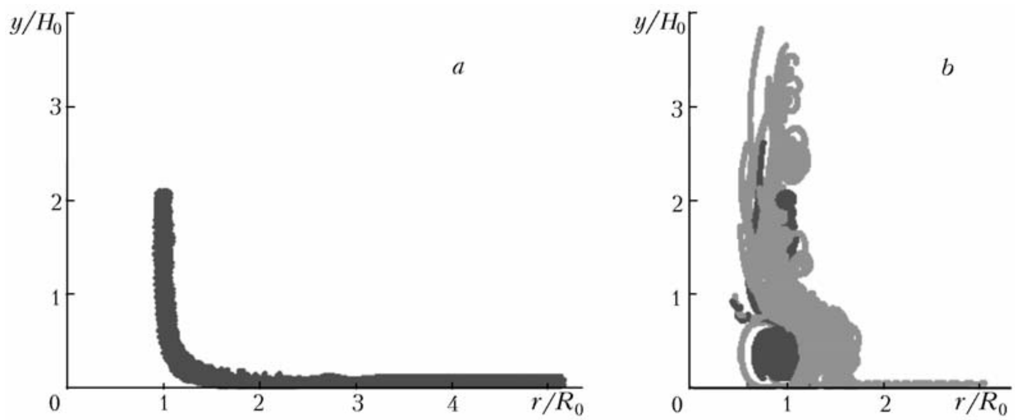

Fig. 3. Incidence of a vortex ring bundle on a flat shield: a) ideal fluid; b) allowance for the medium's viscosity, the turbulent boundary layer on the shield (trajectories of vortex motion), at the left, position of discrete vortices at a fixed instant of time (Re = 10^6). Dark portions correspond to the position of primary vortices; light portions correspond to that of secondary vortices.

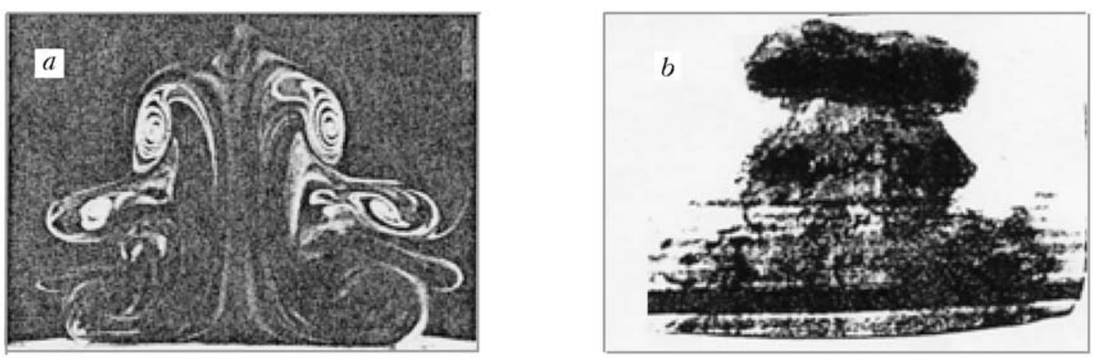

Fig. 4. Visualization of the incidence of a vortex ring bundle on a flat shield (experiments): a) Re = V_0R_0/ν = 930 and b) Re = 1500.

(c_f = 0) and determine the value of the circulation Γ_s of the vortex at the point of separation and the velocity of its downwash. The produced secondary vortex is added to the system of primary vortices, and the process is repeated for the following time interval.

The integral method of calculation of the boundary layer in the quasistationary approximation is only used for determination of the parameters of secondary vortices formed on separation of the layer. Next, within the framework of an ideal medium now, we consider the interaction of the secondary and primary vortices. Figure 2 gives the trajectories of the primary and secondary vortices at two instants of time (a) $\bar{t} = tV_0/H_0$ = 75 and b) 150). These figures illustrate the development of the secondary vortices and their action on the traverse displacement of the primary vortices to form a loop-shaped trajectory of their motion. When the secondary vortices make the first circular turn, we have the separation of the secondary vortices near the shield: one part of them moves along the z axis, whereas the other moves to the plane of symmetry, generating positive vortices near the shield. Noteworthy is the fact that once the first loop-shaped deformation of the primary vortex has been formed, the second identical loop appears (see Fig. 2b).

We have obtained earlier the solution of the corresponding axisymmetric problem on self-induced motion of a ring vortex bundle to a flat shield [4, 5] (Fig. 1b and c). In this case we have the interaction of secondary and primary vortices. However, here these vortices are mixed, with the result that instead of a smooth spreading of the fluid along the shield at a certain instant of time, as in the case of an ideal fluid (Fig. 3a), secondary vortices are formed with allowance for the fluid's viscosity and separation; the interaction of the secondary vortices with the primary ones

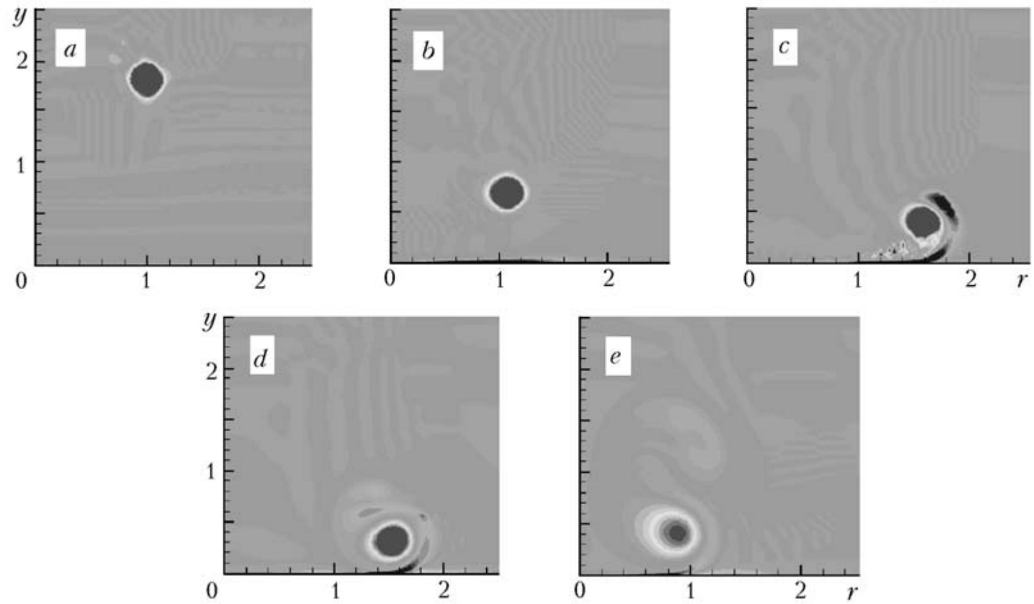

Fig. 5. Successive positions of the ring vortex bundle in its incidence on a flat shield (numerical solution of Navier–Stokes equations [8]).

give rise to reverse flow and then to vertical flow of a mushroom-shaped structure (trajectory of the vortices, Fig. 3b). This flow degenerates with time. Noteworthy is the significant difference of the plane problem from an analogous axisymmetric problem on self-induced motion of a ring vortex bundle to a flat shield [4]. In the axisymmetric case we have the interaction of secondary and primary vortices. However, here these vortices are mixed because of which reverse flow and then ascending flow (which is finally attenuated) is formed at a certain instant of time instead of the smooth spreading of the fluid along the shield.

The above result has been obtained for Reynolds number Re = 10^6 in the case of a turbulent boundary layer. Similar results are also obtained in the case of a laminar boundary layer for Re = 10^3. These data are consistent with the results of visual experiments [6, 7] for Re = 930 (Fig. 4a) and 1500 (Fig. 4b). Here Re = $V_0 R_0 / \nu$, where R_0 and V_0 are respectively the characteristic values of the radius of the axis of the ring vortex bundle and the self-induction velocity with which the solid ring vortex moves to the shield along the axis of symmetry at the initial instant of time [3].

The given example for the axisymmetric problem and the resulting qualitative agreement of the calculated data and experiment demonstrate the efficiency of the presented method of calculation. Publication [4] and the preceding report made by the authors at the seminar of the Institute of Mechanics of the Moscow State University (under the supervision of G. G. Chernyi) initiated investigations of this problem on the basis of numerical solution of unsteady Navier–Stokes equations for laminar flow at small Reynolds numbers [8]. Figure 5 gives successive positions of the ring vortex bundle at different instants of time (a–e) as it approaches the shield. First the ring vortex bundle approaches the shield (Fig. 5a and b); thereafter secondary wall vortex flow with a sign opposite to the sign of the primary ring vortex and its separation are induced on the shield (Fig. 5c), and next the vortex bundle is displaced toward the axis of symmetry under the action of this secondary flow (Fig. 5d and e).

It has been demonstrated in [9] on the basis of the method of viscous vortex domains that allowance for secondary separations makes it possible to describe the deformation of a vortex bundle as it approaches the shield, the development of an ascending flow in the vicinity of the axis of symmetry due to the formation of a high-power secondary vortex of opposite sign, and finally the collapse of the primary vortex because of viscous dissipation.

These results are consistent with our approximate solution. Analogous regularities have been obtained in numerical solution of unsteady Reynolds equations closed using the differential turbulence model for turbulent flow at large Reynolds numbers for the turbulent regime [10].

Conclusions. The time of calculation of one variant of the problem in the enumerated methods is as follows: our approximate solution, 10–15 min, the method of viscous vortex domains, 1 h, and the numerical solution of Reynolds equations, 10–12 h.

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NOTATION

c_f, coefficient of surface friction; H, shape parameter of the boundary layer; H_0, height of location of the rectilinear or ring vortex bundle above the shield at the initial instant of time, m; r, radial coordinate of axisymmetric motion, m; r_0, radius of the core of the rectilinear vortex bundle and accordingly the ring vortex bundle, m; R_0, radius of the axis of the ring vortex, m; Re, Reynolds number; t, time, sec; $\bar{t}$, dimensionless time; V_0, velocity induced by the first rectilinear vortex bundle on the axis of the second, self-induced velocity on the axis of the ring vortex bundle along the vertical axis at the initial instant of time, m/sec; w_z, velocity in the boundary layer, m/sec; $w_{z\delta}$, velocity at the external boundary of the boundary layer, m/sec; y, transverse coordinate, m/sec; z, longitudinal coordinate, m/sec; Δt, time-integration step, sec; δ_1, displacement thickness of the boundary layer, m; δ_2, momentum thickness of the boundary layer, m; Γ_0, circulation of the vortex bundle at the initial instant of time, m^2/sec; Γ_s, circulation of the separated vortex, m^2/sec; ν, coefficient of kinematic viscosity of the fluid, m^2/sec; τ_w, surface friction. Subscripts: 0, initial position; f, friction; s, separation parameter.

REFERENCES

1. S. M. Belotserkovskii, V. N. Kotovskii, M. I. Nisht, and R. M. Fedorov, *Mathematical Simulation of Plane-Parallel Separation Flow Around Bodies* [in Russian], Nauka, Moscow (1988).
2. Al. S. Belotserkovskii and A. S. Ginevskii, Numerical simulation of the vortex wake of an aircraft at take-off–landing regimes, *Dokl. Ross. Akad. Nauk*, **380**, No. 6, 761–764 (2001).
3. K. K. Fedyaevskii, A. S. Ginevskii, and A. V. Kolesnikov, *Calculation of a Turbulent Boundary Layer of an Incompressible Liquid* [in Russian], Sudostroenie, Leningrad (1973).
4. A. S. Ginevskii, T. V. Pogrebnaya, and S. D. Shipilov, Modeling of the incidence of an annular vortex bundle on a plane solid screen, *Dokl. Ross. Akad. Nauk*, **411**, No. 1, 55–57 (2006).
5. O. G. Goman, V. I. Karplyuk, M. I. Nisht, and A. G. Sudakov (M. I. Nisht Ed.), *Numerical Simulation of Axisymmetric Separation Flows of an Incompressible Liquid* [in Russian], Mashinostroenie, Moscow (1999).
6. A. M. Naguib and M. M. Koochessfahani, On wall-pressure sources associated with the unsteady separation in a vortex-ring/wall interaction, *Phys. Fluids*, **16**, No. 7, 2613–2622 (2004).
7. J. D. A. Walker, C. R. Smith, A. W. Cerra, and T. L. Doligalski, The impact of a vortex ring on a wall, *J. Fluid Mech.*, **181**, 99–140 (1987).
8. N. Nikitin, Finite-difference method for incompressible Navier–Stokes equations in arbitrary orthogonal curvilinear coordinates, *J. Comput. Phys.*, **217**, No. 2, 759–781 (2006).
9. P. R. Andronov, S. V. Guvernyuk, and G. Ya. Dynnikova, *Vortical Methods of Calculation of Nonstationary Hydrodynamic Loads* [in Russian], Inst. Mekhaniki MGU, Moscow (2006).
10. N. A. Vladimirova, Dynamics of the viscous interaction of a ring vortex with a flat screen, *Inzh.-Fiz. Zh.*, **81**, No. 1, 184–190 (2008).